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## Technical Note

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### Coherent Overlaying – A Signature Enhancement Processing Technique

5 May 1975

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FOR THE COMMANDER

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COHERENT OVERLAYING – A SIGNATURE ENHANCEMENT  
PROCESSING TECHNIQUE

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*Group 96*

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## ABSTRACT

A technique called coherent overlaying has been developed for improving the signal-to-noise ratio in plots of radar return vs. time. The essential feature of the technique is a phase corrected summation of pulses which have been scattered by the same point on a target. Several test cases are discussed and the time signature of ATS-3 which was too weak to be extracted by regular processing of Millstone Hill radar data is presented.

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## COHERENT OVERLAYING -- A SIGNATURE ENHANCEMENT PROCESSING TECHNIQUE

### I. Introduction

A plot of target return versus time is a very useful output from radar observations of an object. In the simplest of systems it may be the basis for detecting targets. In other systems it may be used to distinguish one target from another. Then again, having such an output in his possession, an analyst can infer target structure or combine his knowledge of the target structure with the observed returned power to infer target motion. In many cases, however, the target return is sufficiently weak that the pulse by pulse signal-to-noise ratio is too low to produce an identifiable signature.

In such cases if the return is periodic, one might combine the data from a number of cycles of target motion to produce a composite graph with greater definition than the single cycle data. We frequently do this using incoherent addition of the returned pulses coming from the same point on the target. If the radar is coherent, there are processing options which can improve the definition of the signature still more. One such option we call coherent overlaying. It is a technique of improving the signal-to-noise ratio by coherently summing pulses taken from subsequent cycles of target motion. Figure 1 illustrates the coherent overlaying technique. The essential feature of the technique is a phase corrected coherent summation of pulses which have been scattered by the same point on the target.

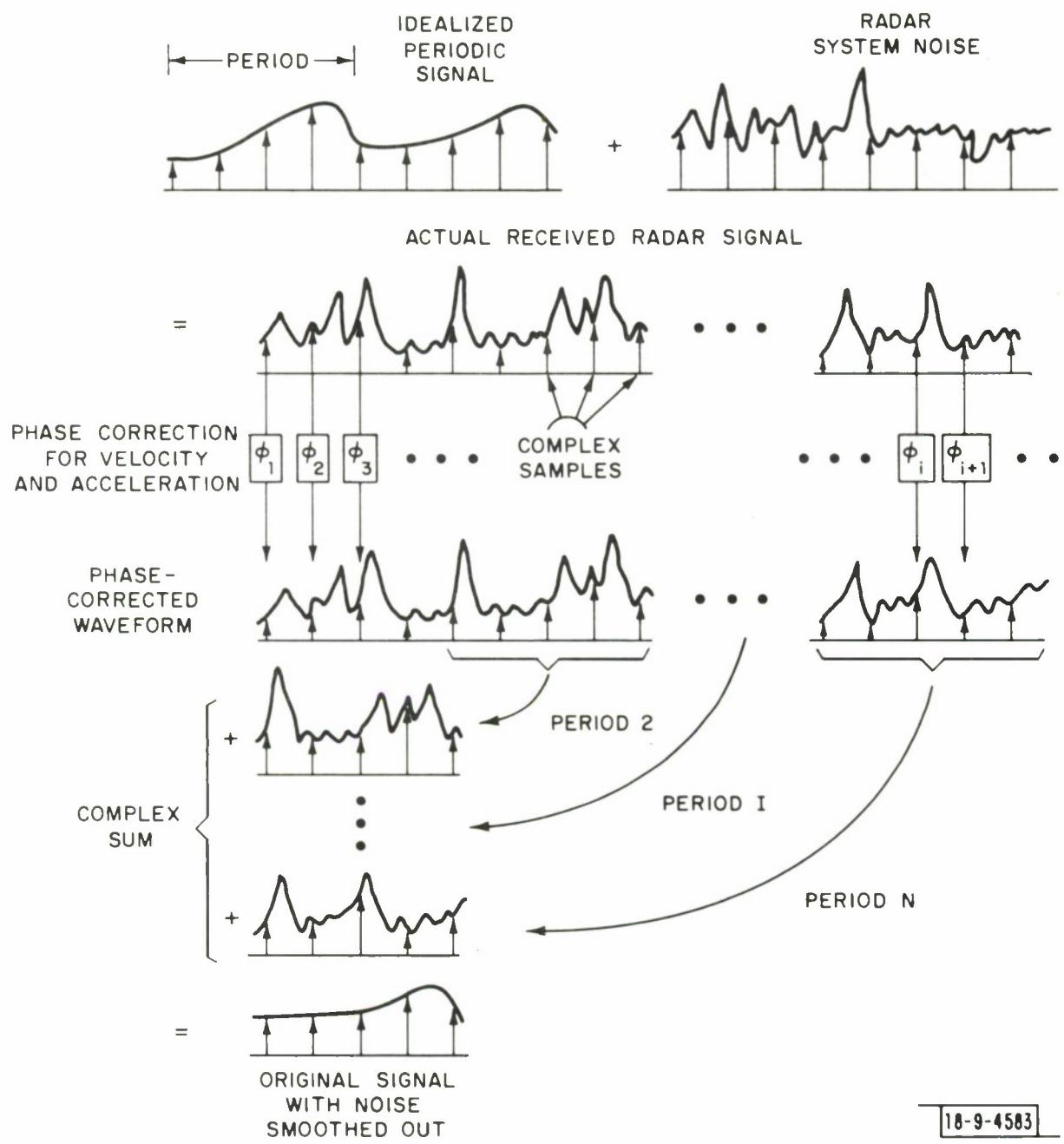


Fig. 1. Coherent overlay procedure.

There are two components of the phase variation in the recorded signal. One is from the motion of the target about its center of mass and the second is either from motion of the center of mass with respect to the radar or the residual left after a real time correction for motion of the center of mass. If the second term is not corrected for, pulses scattered by the same point on the target may not add in phase. The phase correction in the coherent overlay allows for removal of this term in the recorded data. It is accomplished by observing the location and motion of a line in the target spectrum and using the corresponding derivatives of phase for the correction terms. If done correctly, there will be negligible phase change from a given point in one cycle to the corresponding point in the next. Consequently such points will add in phase to give a plot of power versus time which represents a single cycle of target motion but has a signal-to-noise ratio much greater than in the single cycle plots.

## II. Mathematical Background

Suppose the recorded pulses from each cycle of target motion are placed in N buffers of length M and let  $\bar{Z}_{ij}$  be the complex received signal which has already been phase-corrected for target motion and ionospheric effects - i is the index in the buffer and j is the buffer number. We define the incoherent sum by

$$P_j^I = \frac{1}{N} \sum_{i=1}^N \left\| \bar{z}_{ij} \right\|^2$$

and the coherent sum by

$$P_j^C = \frac{1}{N^2} \left\| \sum_{i=1}^N \bar{Z}_{ij} \right\|^2$$

The statistics of each sum with Gaussian noise being processed are of interest.

It is assumed  $X_{ij}$ , the real, and  $Y_{ij}$ , the imaginary, parts of  $\bar{Z}_{ij}$  are independent, normal random variables each with zero mean and variance  $\sigma^2$ . The distribution of  $P_j^I$  is now found. For a given  $i$ ,  $\left\| Z_{ij} \right\|^2 = X_{ij}^2 + Y_{ij}^2$  has the exponential distribution

$$f_z(z) = \frac{1}{2\sigma^2} e^{-z/2\sigma^2} U(z) \quad \text{where } U(z) \text{ is the unit step function.}$$

Transforming  $f_z(z)$  gives the characteristic function

$$\phi(\omega) = (1 - j2\omega\sigma^2)^{-1}$$

Summing  $N$  terms results in a characteristic function of

$$\phi_N(\omega) = (1 - j2\omega\sigma^2)^{-N}$$

This is the characteristic function of a chi-square distribution with  $2N$  degrees of freedom and the distribution of the sum is

$$f(x) = \frac{x^{N-1}}{(2\sigma^2)^N \Gamma(N)} e^{-x/2\sigma^2}$$

The distribution function of  $P_j^I$  follows from  $f(Y) = \frac{1}{|a|} f\left(\frac{Y}{a}\right)$  where  $Y = ax$  and  $a$  is constant to be

$$f_{P_j^I}(Y) = \frac{Y^{N-1}}{\left(\frac{2\sigma^2}{N}\right)^N \Gamma(N)} e^{-YN/2\sigma^2} U(Y)$$

Therefore,  $P_j^I$  is chi-squared distributed with mean  $2\sigma^2$ , standard deviation  $2\sigma^2/\sqrt{N}$  and  $2N$  degrees of freedom.

The distribution for  $P_j^C$ , the coherent sum, is found as follows. First note the distributions of the sums of the real and imaginary terms taken separately are simply

$$f(a) = \frac{1}{\sqrt{\frac{2\pi\sigma^2}{N}}} e^{-Na^2/2\sigma^2} \quad \text{for } a = X, Y$$

The square of each sum  $W = a^2$  has the distribution

$$f(W) = \frac{1}{\sqrt{\frac{2\pi\sigma^2}{N}}} e^{-NW/2\sigma^2} U(W)$$

The result for the distribution of  $P_j^C$  follows as the convolution of the densities of  $\frac{1}{N} \left( \sum X_{ij} \right)^2$  and  $\frac{1}{N} \left( \sum Y_{ij} \right)^2$ . Taking the product of the characteristic functions and transforming back gives:

$$f_{P_j^C}(Y) = \frac{N}{2\sigma^2} e^{-YN/2\sigma^2} U(Y)$$

$P_j^C$  thus has an exponential distribution with mean  $2\sigma^2/N$  and variance  $2\sigma^2/N$ .

Note the mean noise power is a factor of  $N$  lower than in the incoherent case.

In the case where signal power alone is present it is readily seen the coherent and incoherent sums give the same result. It follows that the effect of coherent overlaying is to reduce the noise power by N which in effect is an increase of N in the signal-to-noise ratio over the incoherent case.

The relevant statistics of the coherent and incoherent overlays are summarized in Table 1.

### III. Candidate Targets and Implementation Considerations

For the coherent overlay procedure to make sense the target return must be periodic. Beyond this it is required the total duration of data to be processed be less than the time over which the radar system including propagation path is coherent. Of course, the radar PRF should be sufficient to sample all significant variation in cross section. For the time plots to be meaningful one should make the further stipulation that the resultant signal-to-noise ratio be high enough to produce useful results. Depending on the application, the minimum post-processing signal-to-noise ratio is usually around 7 to 10 dB.

All the radars which we have used for satellite observations have been limited in their coherent processing capability by the minimum frequency detent in their local oscillators\*. The radars and the corresponding maximum coherent

\* In all cases we have been able to perform coherent integration over the interval given by the inverse of the oscillator detent. It is likely that conditions exist, particularly for the lower frequencies, during which ionospheric conditions preclude such integration. However, in the data we have examined we have not yet been hampered by such a case.

TABLE 1

## COMPARISON OF COHERENT AND INCOHERENT OVERLAYS

	Mathematical Form (jth bin)	Probability Distribution with Gaussian Noise	Statistics of Overlays
Coherent	$P_j^C = \frac{1}{N^2} \left\  \sum_{i=1}^N \bar{z}_{ij} \right\ ^2$	$f_{P_j^C}(Y) = \frac{N}{2\sigma^2} e^{-YN/2\sigma^2} U(Y)$	mean = $2\sigma^2/N$ std. dev. = $2\sigma^2/N$
Incoherent	$P_j^I = \frac{1}{N} \sum_{i=1}^N \left\  \bar{z}_{ij} \right\ ^2$	$f_{P_j^I}(Y) = \frac{Y^{N-1}}{\left(\frac{2\sigma^2}{N}\right)^N} \frac{e^{-YN/2\sigma^2}}{\Gamma(N)} U(Y)$	mean = $2\sigma^2$ std. dev. = $2\sigma^2/\sqrt{N}$

$\bar{z}_{ij}$  = complex received signal in a designated bin of buffer i

processing times are shown in Table 2. Assuming single cycles of target motion are summed, Fig. 2 has been constructed to show the size of cross section that will result in a signal-to-noise ratio of 10 when M such cycles from a target at synchronous range are coherently overlaid.

TABLE 2  
LIMITATION ON COHERENT PROCESSING TIME

<u>Radar</u>	<u>Frequency Detent Limited Coherent Processing Time</u>
Arecibo (430 MHz)	100 sec
Millstone (1290 MHz)	1000 sec
Haystack (7840 MHz)	100 sec

The phase correction terms are derived by taking Fast Fourier Transforms (FFT's) of the data being processed. Suppose the total duration of the data is N points\* and in an N point transform the center of the spectrum is k points off the zero frequency bin ( $= (N/2) + 1$ ). The displacement from zero represents a velocity error which must be corrected before the overlay is attempted. In particular each bin off zero represents a velocity increment of  $\Delta v = f_d \lambda$ , where  $f_d$  = doppler frequency,  $\lambda$  = radar wavelength. But  $f_d = 1/NT$ , where T = interpulse

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\* The discussion treats N as even. In any event one could drop or add a pulse to insure this. Note that here N represents an integral number of periods of target motion. This condition is relaxed subsequently.

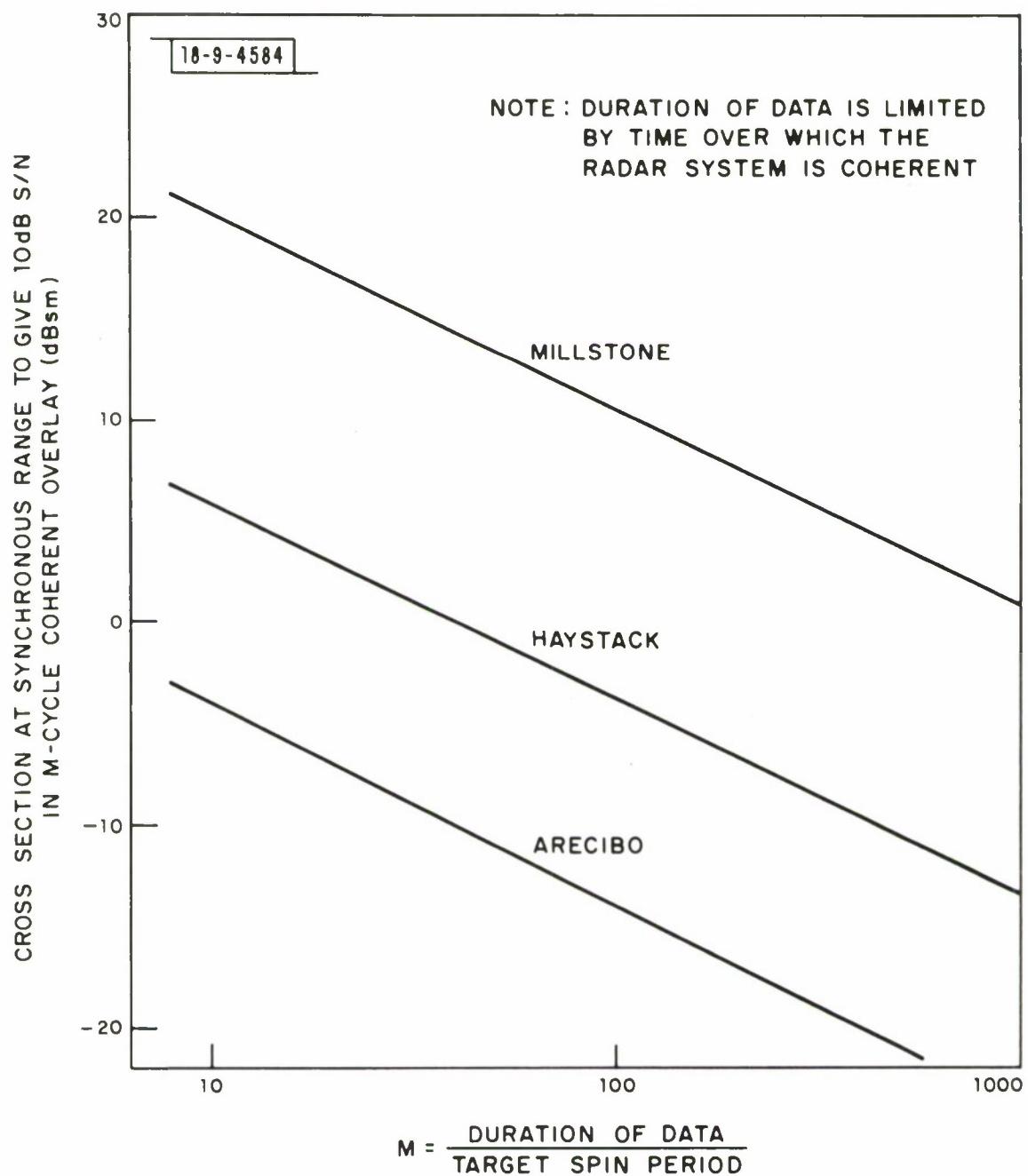


Fig. 2. Minimum cross-sections seen in M cycle coherent overlay.

period. So  $v = \lambda/NT$ . Over one pulse this velocity results in a phase change of

$$\frac{360}{N} \text{ degrees/pulse.}$$

To correct for  $k$  bins off the center we multiply  $360/N$  by  $k$ . When this phase is subtracted from the original data the resultant will have no net phase change over the  $N$  pulse interval.

It may be that there is acceleration also present in the original data. This would be noticed by the signal spreading out over several adjacent bins of the FFT. To size the corrections here one can take consecutive  $N/2$  point transforms of the data. In the presence of acceleration a designated peak will move by  $n$  bins from one transform to the next. The corresponding change in doppler frequency is

$$\Delta f_d = \frac{2n}{NT}$$

The time over which the change takes place is  $\Delta t = NT/2$  so

$$\frac{\Delta f_d}{\Delta t} = \frac{4n}{(NT)^2}$$

The corresponding acceleration is  $4n \lambda/(NT)^2$  and expressing the result as above in terms of degrees and interpulse periods, gives an acceleration correction of

$$\frac{1440}{N^2} n \text{ degrees/(pulse)}^2$$

This correction is then applied to the N point transform and the resulting transform can be tweaked by using acceleration variations of  $360n/N^2$  about the initial value.

After the velocity and acceleration corrections are applied pulse by pulse to the data, the data stream is ready to be broken into periods and overlaid. At this point it is usually necessary to deal with a non-integral number of pulses in each period of target return.

The simplest way to handle a non-integral number of pulses in the period is to use several cycles as the basic period. Frequently, a basic period of more than one cycle can be selected so the larger period contains an integral number of pulses and subsequent sets of data match exactly. Unfortunately, the maximum time span over which data can be used for a given overlay is limited by system/ionospheric coherence, so the maximum coherent gain achievable is decreased by the number of cycles in the basic processing interval.

In order to better use the available data an algorithm for selectively dropping pulses has been implemented. The algorithm treats successive periods to be added as buffers and fills and coherently adds successive buffers until it calculates a slip of over one target cycle plus one half an interpulse period has occurred between the present buffer and either the first pulse in the first buffer or the first pulse in the last buffer after which the correction was made. At this point the next pulse in the data stream is skipped and the addition continues. The maximum phase error is bounded by the phase change of the target over

one-half an interpulse period; as long as this change is not too great, the technique is viable. To date it is the only technique we have used.

#### IV. Testing the Overlay Algorithm

A major problem in advocating a technique that is meant to produce results that are achievable no other way is in offering a convincing demonstration that the underlying structure is both sound and correctly implemented. To validate the coherent overlay procedure, three checks have been made.

##### a) Artificial Signal and Strong Actual Signal

In the first tests a ramp input was constructed and successive cycles summed. The results were compared with independently calculated performance and excellent agreement obtained as both phase shifts and the number of points in the cycle (including fractional parts) were varied. An equally good comparison occurred between a signal of very high signal-to-noise ratio when summed both incoherently and with a coherent overlay. In this case one would expect equal results in that both techniques handle signal equally; it is in their noise smoothing abilities that they differ.

##### b) Noise Processing

Samples of noise were processed with the coherent overlay as a further check on performance. Figure 3 shows a sample of noise data recorded at Arecibo on 13 Feb. 1974. Figures 4 and 5 show the effect of processing 50 such sets of noise data with incoherent and coherent overlays. In the incoherent case, we

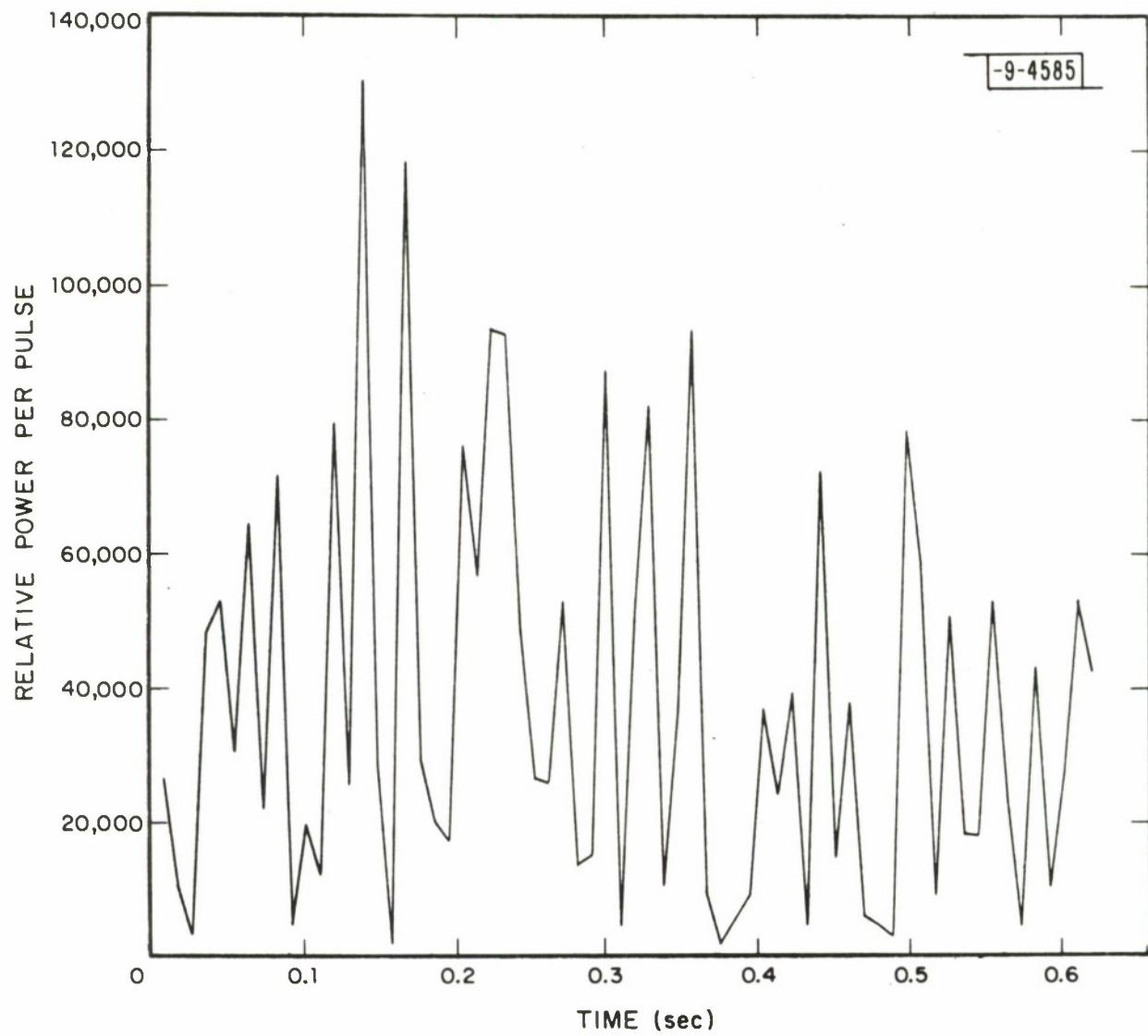


Fig. 3. Pulse by pulse plot of noise recorded at Arecibo 13 February 1974.

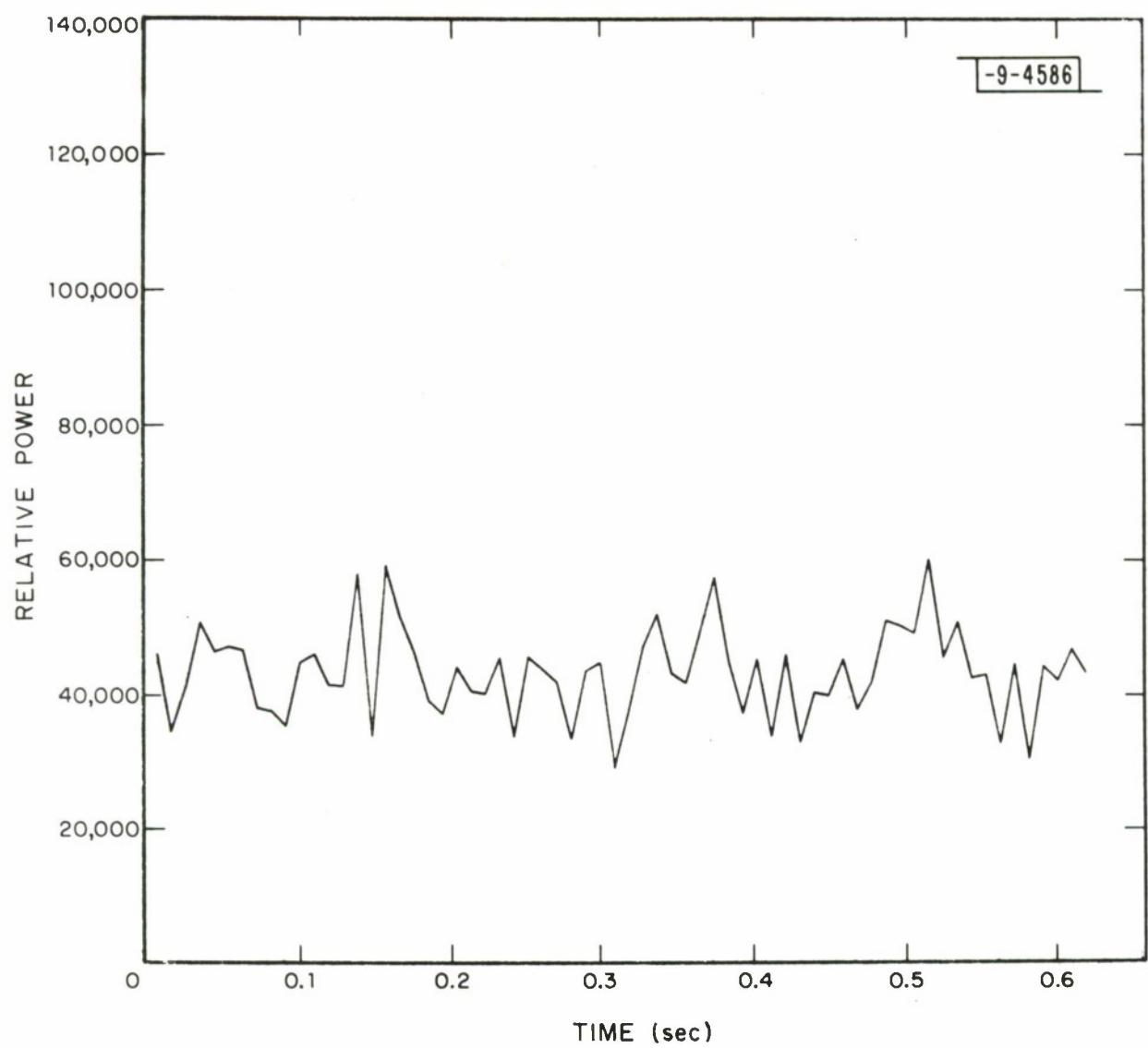


Fig. 4. Incoherent overlay of 50 sets of 66 pulses each of noise data as shown in Fig. 3.

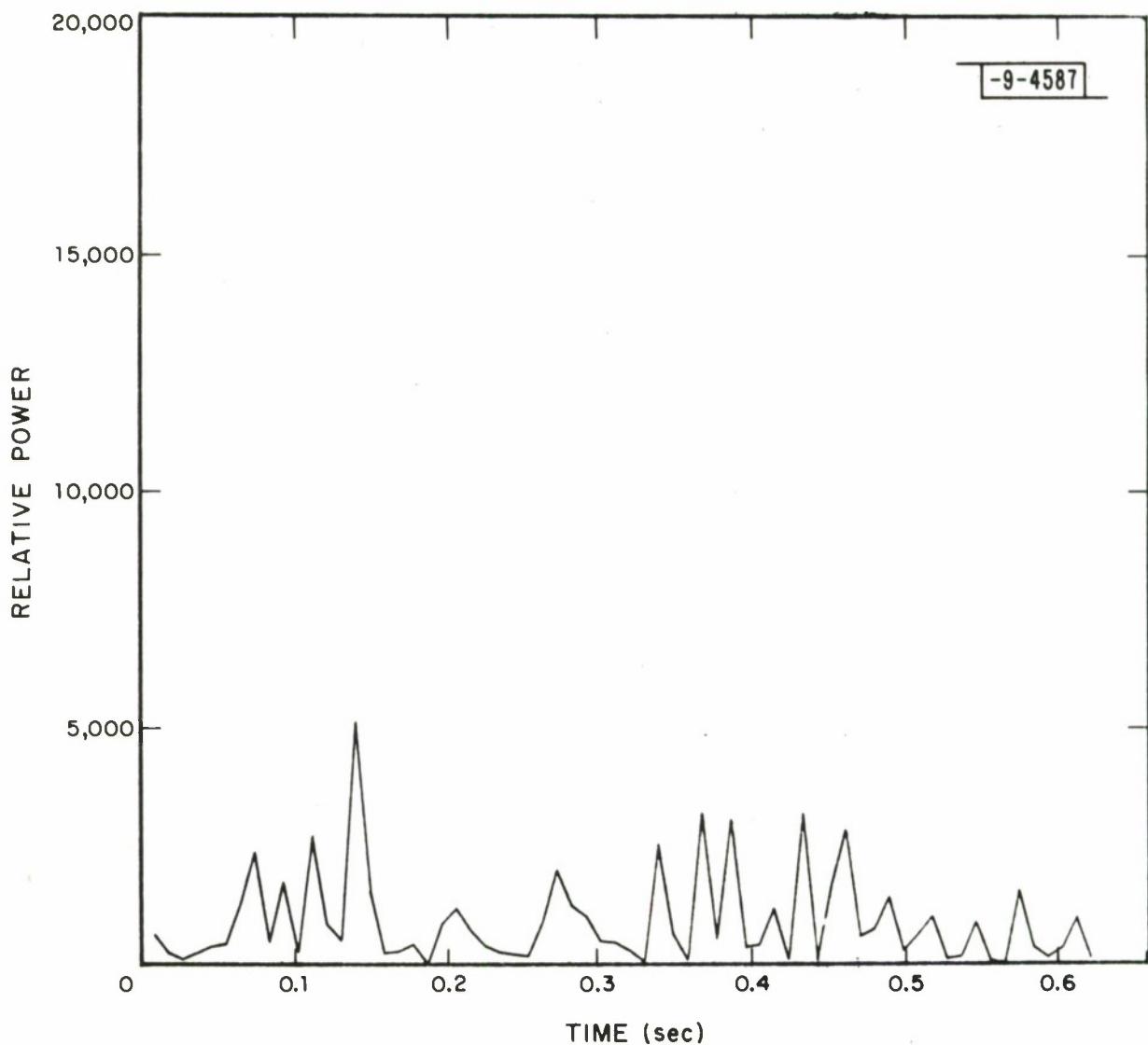


Fig. 5. Coherent overlay of 50 sets of 66 pulses each of noise data as shown in Fig. 3.

expect the mean noise will equal the mean of the individual samples. This value is 43,400. Its standard deviation is computed to be  $43,400/\sqrt{50} = 6140$ . This value certainly fits what is observed in Figure 4.

In the coherent overlay we expect a mean and standard deviation of  $43,400/50 = 870$  for each sample. The mean of the samples shown in Figure 5 is 910. Since this is the mean of 66 samples, a standard deviation of  $1/\sqrt{66}$  of the original sample mean is expected. The computed mean is within  $.4\sigma$  of the expected mean and we conclude the overlay is performing as it should.

Several other similar cases have been successfully run. The coherent overlays give every indication of successfully processing noise.

### c) Signal in Noise Processing

The most convincing demonstrations of the proper operation of the coherent overlays came from several cases in which it has been possible to compare results of overlaying with better estimates of the signal which were derived from cases having a higher signal-to-noise ratio.

On 10 Feb. 1975, the Haystack observations were hampered by difficulties with the klystrons which limited the peak power to about 10 dB below its normal value. In particular, the incoherent average of 2 cycles of power vs. time for 4F3, Fig. 6, is much noisier than the corresponding average Fig. 7 from data taken on 3 Oct. 1974 at which time the power was close to normal. Coherently overlaying 10 sets of 2 cycles of data results in Fig. 8. The results compare well with the

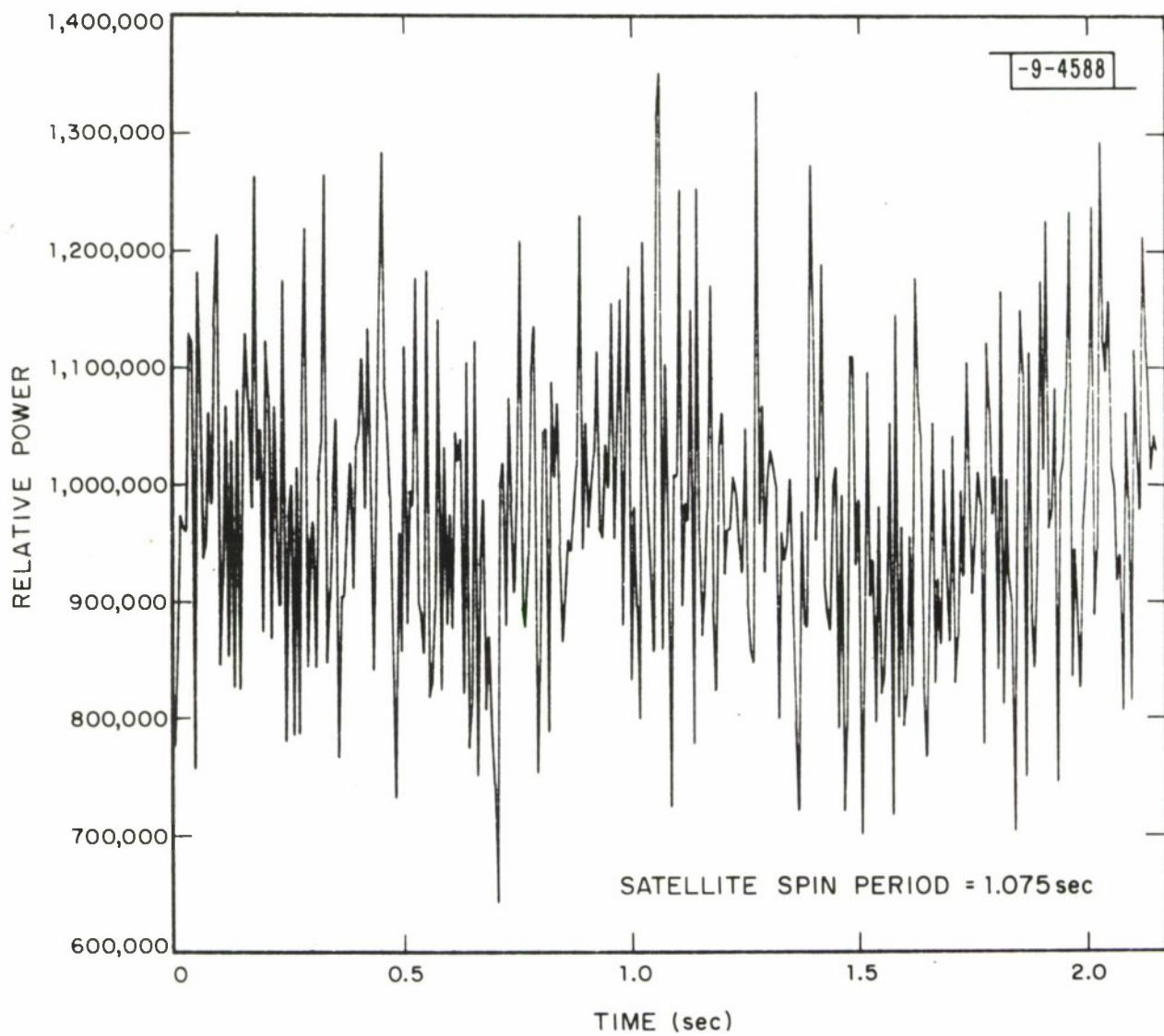


Fig. 6. Incoherent overlay of 10 sets of two satellite periods each of 4F3 data taken 10 February 1975.

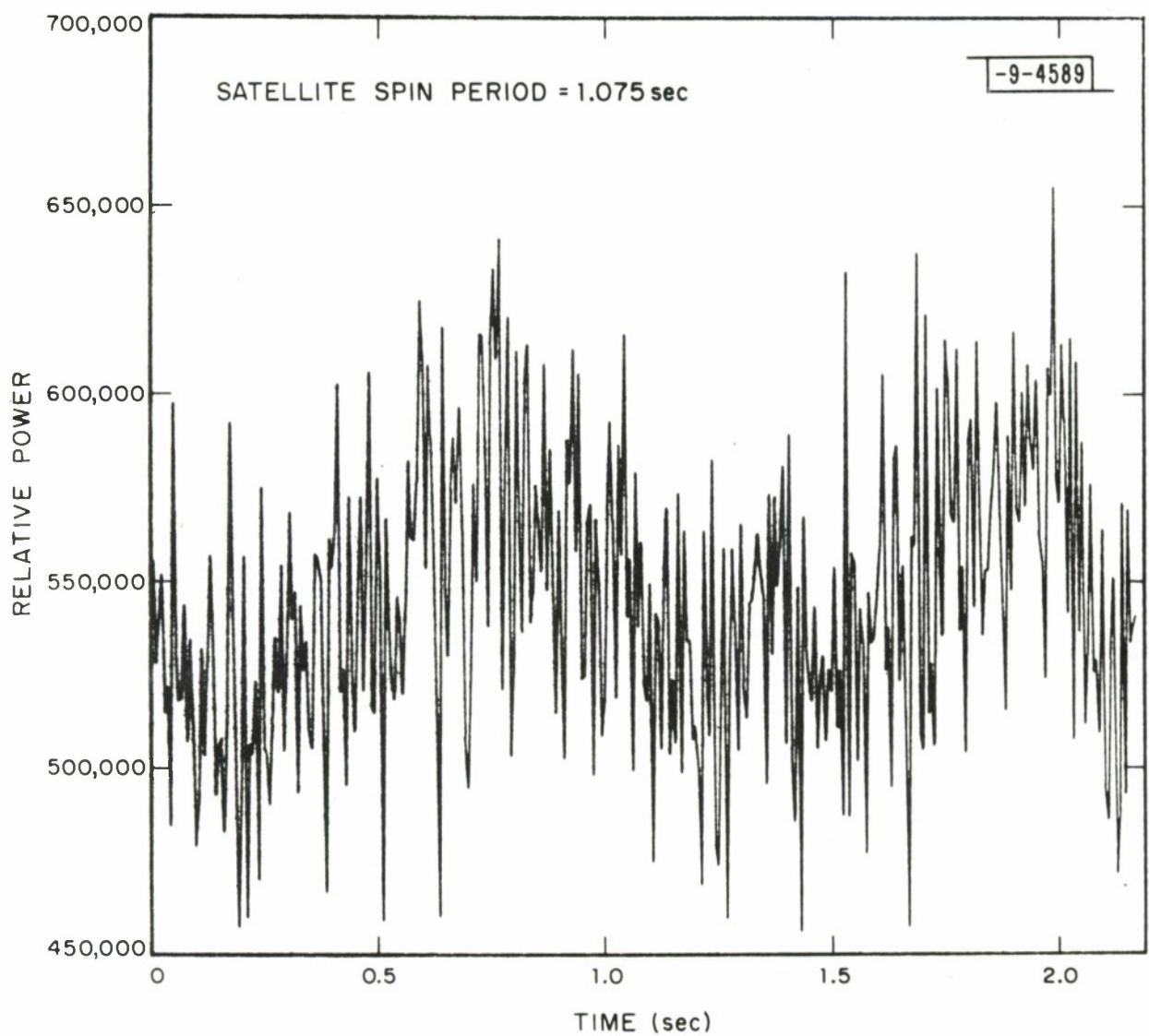


Fig. 7. Incoherent overlay of 5 sets of two satellite periods each of 4F3 data taken 3 October 1974.

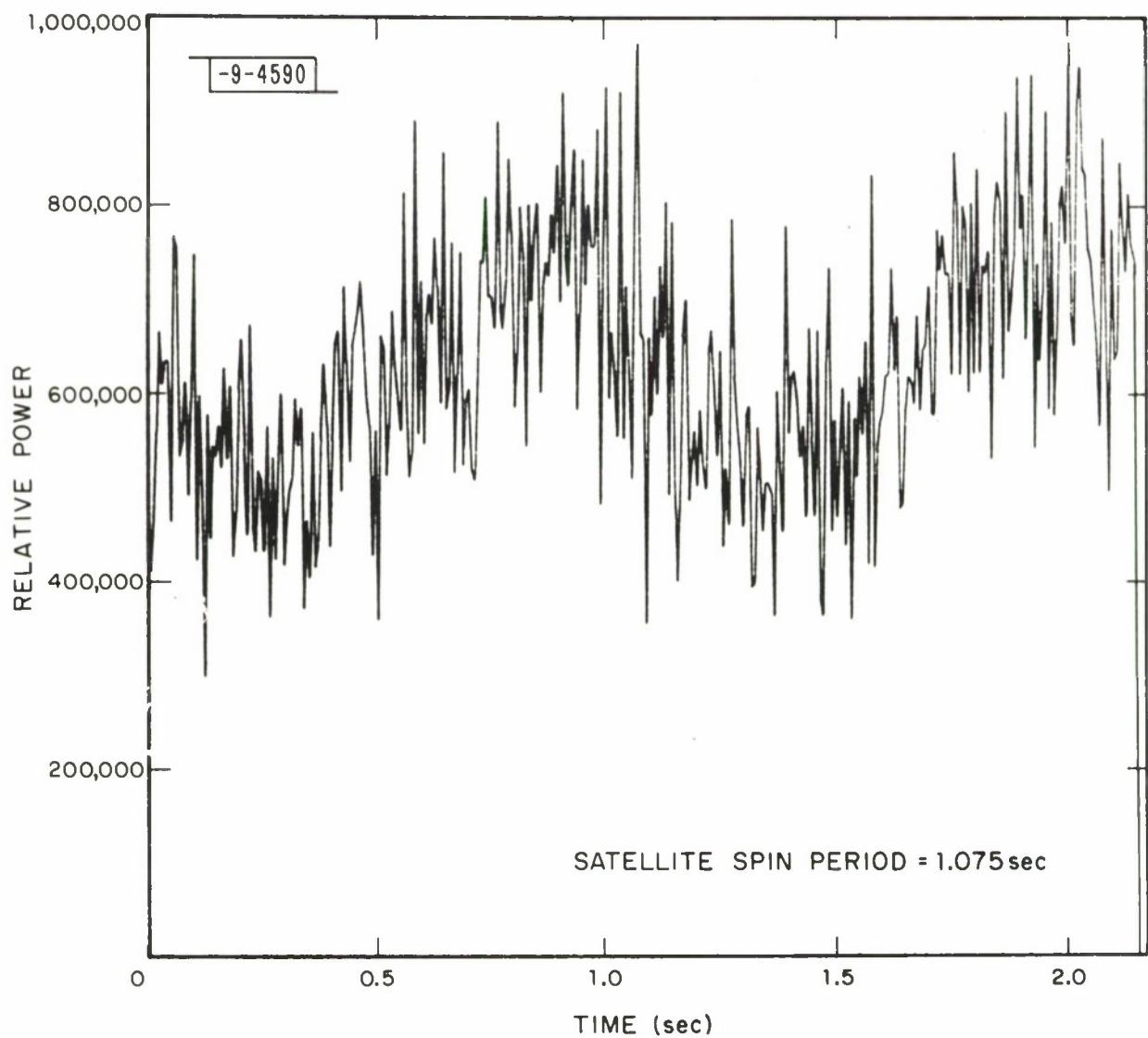


Fig. 8. Coherent overlay of 10 sets of two satellite periods each of 4F3 data taken 10 February 1975.

stronger data of Fig. 7. The 10 dB loss in transmitted power has been recovered by the coherent processing.

The second test of the overlay procedure was performed using data on ATS-3 taken at Arecibo. When the data were recorded a number of samples of matched filter output were taken as shown in Fig. 9. The effect of the sampling was to provide a channel with strong signal (C3) and one with weak signal (C1). Fig. 10 shows the coherent overlay of 50 cycles of data from C1 on which the incoherent overlay of strong signal from C3 has been superimposed. The average signal-to-noise ratio was -10 dB in C1 before overlaying. After the overlaying of 50 periods it should be 7 dB. The result actually obtained compares favorably with the C3 results which show a signal-to-noise ratio of 15 dB. Only the third peak of the overlay fails to match well with the "actual" signal of C1.

#### V. Time Signature of ATS-3 from Millstone

The major achievement to date in using the coherent overlaying procedure has been the production of a time signature for NASA satellite ATS-3 from the Millstone Hill L-band radar. Radar parameters for the time of observation are shown in Table 3. In the unprocessed recorded data the average per pulse signal-to-noise ratio was measured as -20 dB.

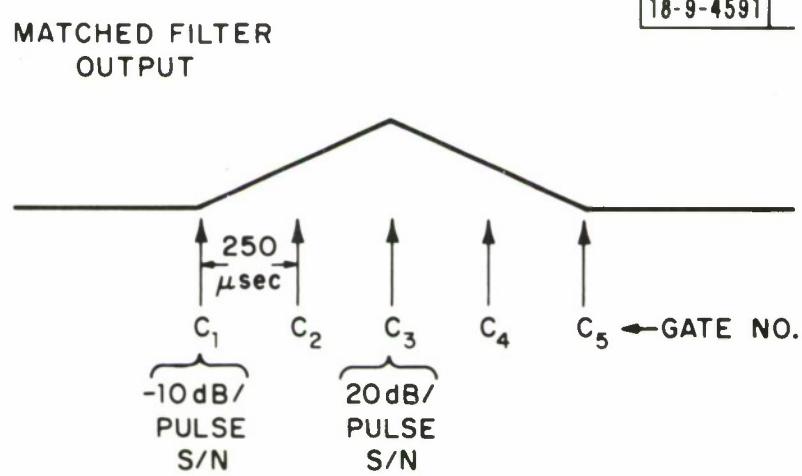


Fig. 9. ATS-3 Arecibo coherent overlay test case.

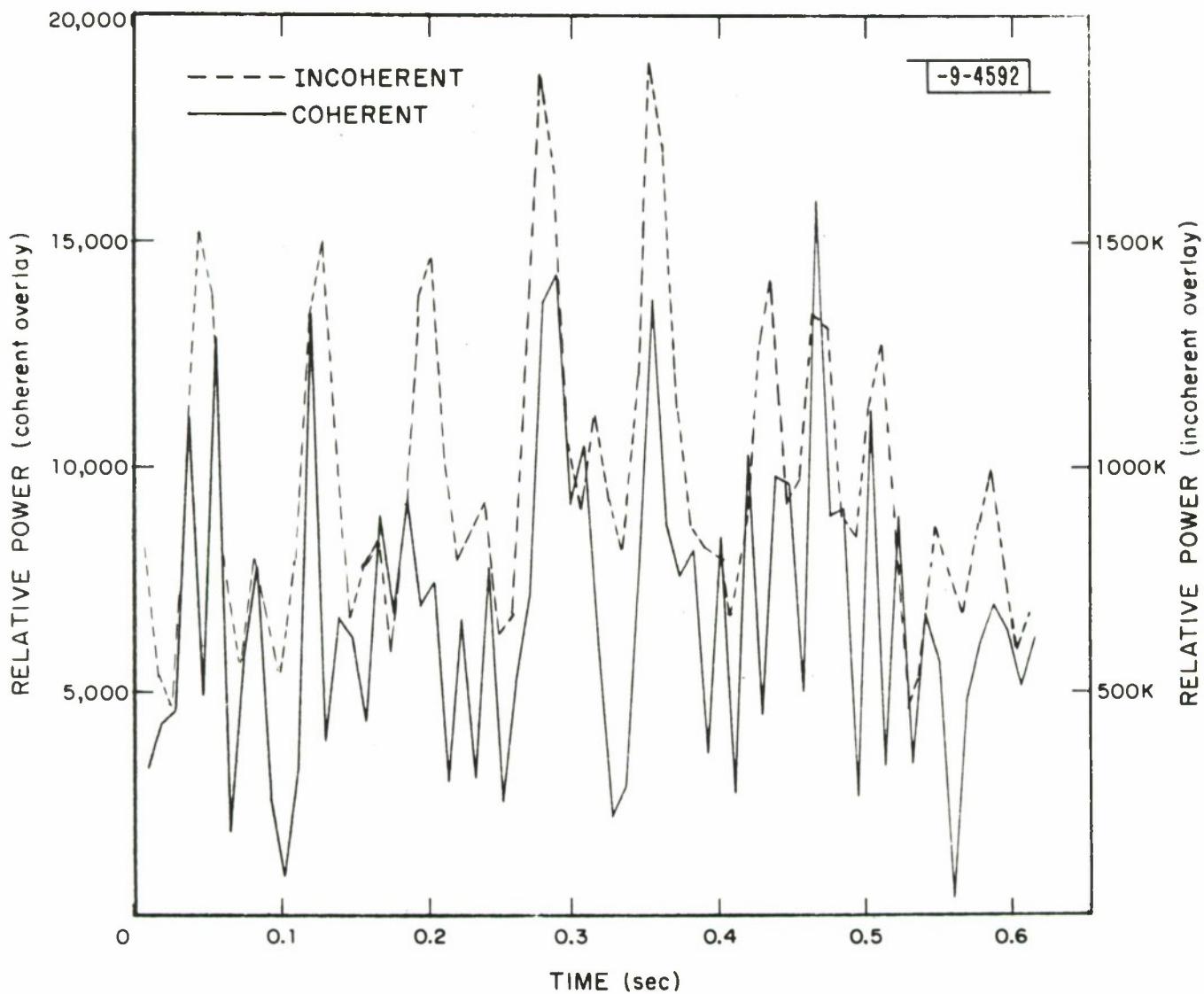


Fig. 10. Comparison of coherent overlay with incoherent data in a high S/N channel.

TABLE 3  
MILLSTONE RADAR PARAMETERS DURING 3 JANUARY 1975 OBSERVATION OF ATS-3

Peak Power	3.0 Mw
Pulse Length	2 msec
PRF	16.67 Hz
Wavelength	23 cm
Antenna Gain	46.6 dB
System Temperature	150° K
System Losses	3.8 dB
Single Pulse S/N on $1m^2$ Target at 38,500 km	-15.1 dB

Coherent and incoherent overlays of the data were made. The results are shown in Figs. 11 and 12. The plots are power vs. time for three cycles of target motion. To produce each plot 500 sets of three cycles each were overlaid. This represents over 900 seconds of radar data. In the coherent overlay the average signal-to-noise ratio is 4 dB. The improvement of 24 dB is within 3 dB of the theoretical 27 dB. The resultant signature agrees well with what was expected based on theoretical models derived from X-band and UHF observations of the object.

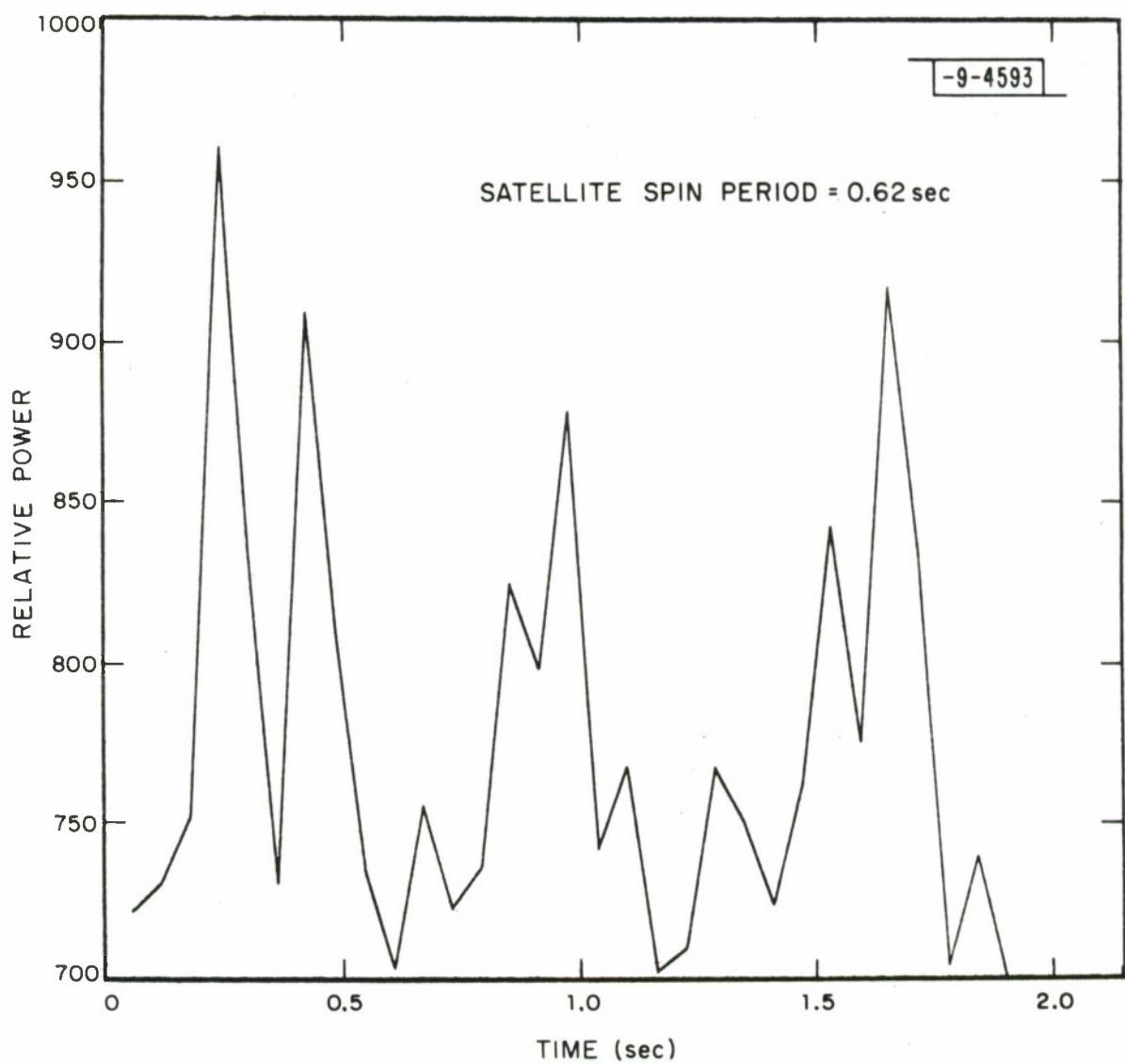


Fig. 11. Coherent overlay formed with ATS-3 data from Millstone 3 January 1975.

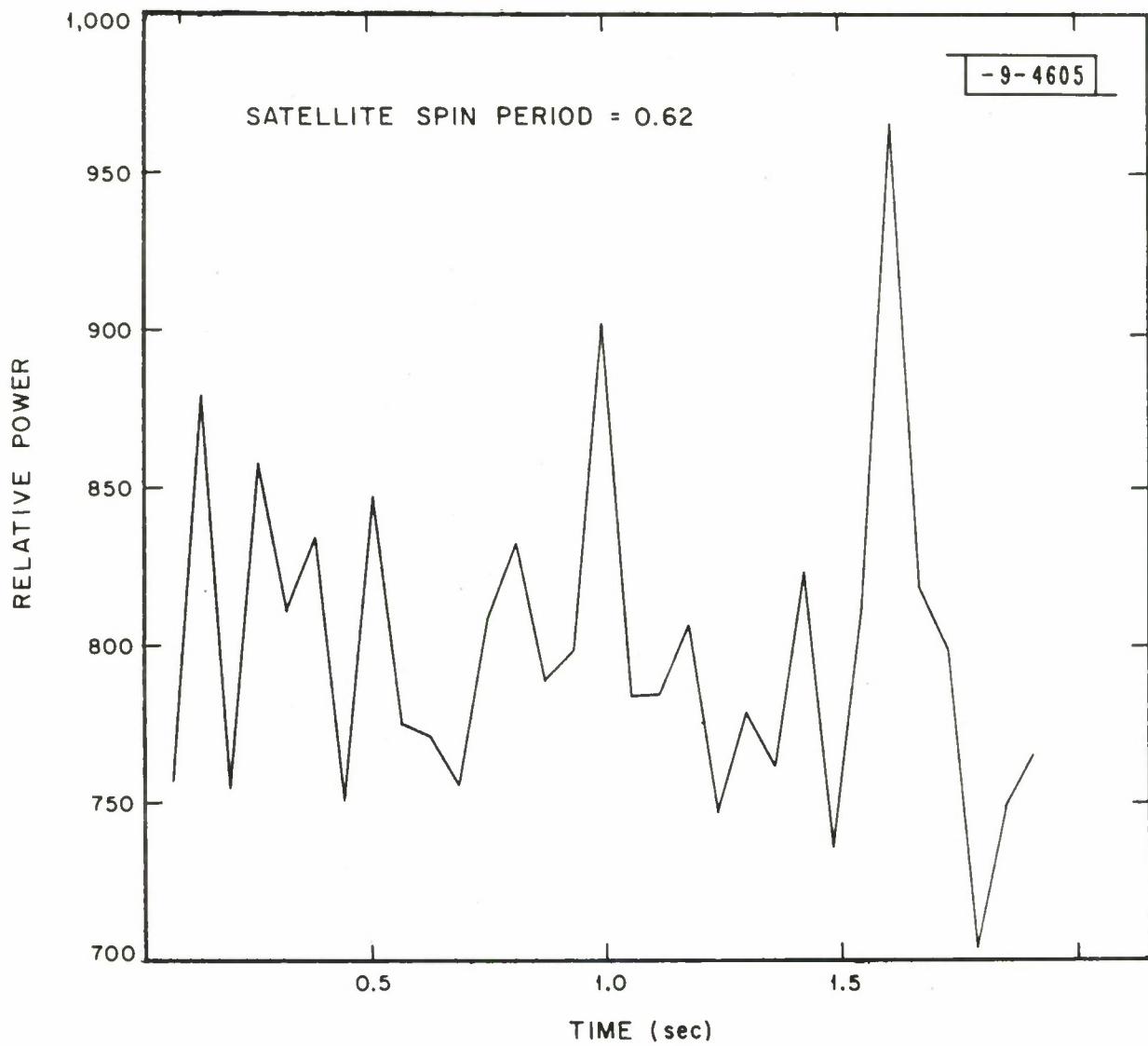


Fig. 12. Incoherent overlay formed with ATS-3 data from Millstone 3 January 1975.

The incoherent overlay does not, of course, increase the signal-to-noise ratio. It merely allows better definition of it by reducing the variance of the noise. In the incoherent overlay the major spikes of the coherent overlay are seen but nowhere as clearly as in the coherent case.

The use of the coherent overlay has made possible the extraction of a time signature from data which are of too low a signal-to-noise ratio for regular processing to be meaningful. Work is continuing on producing signatures for other targets as well as considering near real time implementation of the technique.

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